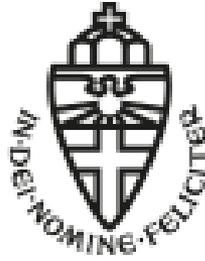


Radboud Universiteit



Selected problems of two-dimensional magnetism

Mikhail Katsnelson



Institute for Molecules and Materials

Outline

- I. Mermin-Wagner, short-range order vs long-range order, and all that
- II. False start: K_2CuF_4
- III. CrX_3 : ferrons, magnetic anisotropy, excitons
- IV. CrSBr : highly anisotropic, tunable 2D
- V. Fe_3GeTe_2 : interplay of phonons and magnetism

Peculiarities of 2D magnets

Soft Goldstone modes in 1D and 2D result in divergent corrections to magnetization which naively means destruction of magnetically ordered state. Rigorous result: the absence of long-range order in 2D Heisenberg model at finite temperatures (Mermin and Wagner, 1966). However, at low temperatures: strong short-range order, exponentially large correlation length (Polyakov, 1975).

Mean-field way to describe the state with short-range order: self-consistent spin wave theory (SSWT), suggested and applied to 2D Heisenberg ferromagnet in different forms by Arovas & Auerbach, 1988; Yoshioka, 1989; Takahashi, 1989.

Easy-axis anisotropy or small interlayer exchange stabilize ordered phase
- what to do?

SSWT of 2D magnetism

PHYSICAL REVIEW B

VOLUME 60, NUMBER 2

1 JULY 1999-II

Self-consistent spin-wave theory of layered Heisenberg magnets

V. Yu. Irkhin, A. A. Katanin, and M. I. Katsnelson

$$H = -\frac{1}{2} \sum_{ij} J_{ij} \mathbf{S}_i \mathbf{S}_j - \frac{1}{2} \eta \sum_{ij} J_{ij} S_i^z S_j^z - D \sum_i (S_i^z)^2$$

Easy-axis anisotropy

$$\eta > 0 \text{ and } D > 0$$

Baryakhtar-Krivoruchko-Jablonsky representation for spin operators

$$S_i^+ = \sqrt{2S} b_i, \quad S_i^z = S - b_i^\dagger b_i - (2S+1) c_i^\dagger c_i, \quad (2)$$

$$S_i^- = \sqrt{2S} \left(b_i^\dagger - \frac{1}{2S} b_i^\dagger b_i^\dagger b_i \right) - \frac{2(2S+1)}{\sqrt{2S}} b_i^\dagger c_i^\dagger c_i,$$

where b_i^\dagger, b_i are the Bose ideal magnon operators, and c_i^\dagger, c_i are the auxiliary pseudofermion operators at the site i which take into account the kinematic interaction of spin waves.

Treat anharmonic (multimagnon) terms in Hartree-Fock-style way

SSWT of 2D magnetism II

Another useful representation of spin operators is the Schwinger-boson representation

$$\mathbf{S}_i = \sum_{\sigma\sigma'} s_{i\sigma}^\dagger \boldsymbol{\sigma}_{\sigma\sigma'} s_{i\sigma'}, \quad (6)$$

where $\boldsymbol{\sigma}$ are the Pauli matrices, $\sigma, \sigma' = \uparrow, \downarrow$, so that

$$S_i^z = \frac{1}{2} (s_{i\uparrow}^\dagger s_{i\uparrow} - s_{i\downarrow}^\dagger s_{i\downarrow}), \quad S_i^+ = s_{i\uparrow}^\dagger s_{i\downarrow}, \quad S_i^- = s_{i\downarrow}^\dagger s_{i\uparrow}. \quad (7)$$

The constraint condition

$$s_{i\uparrow}^\dagger s_{i\uparrow} + s_{i\downarrow}^\dagger s_{i\downarrow} = 2S \quad (8)$$

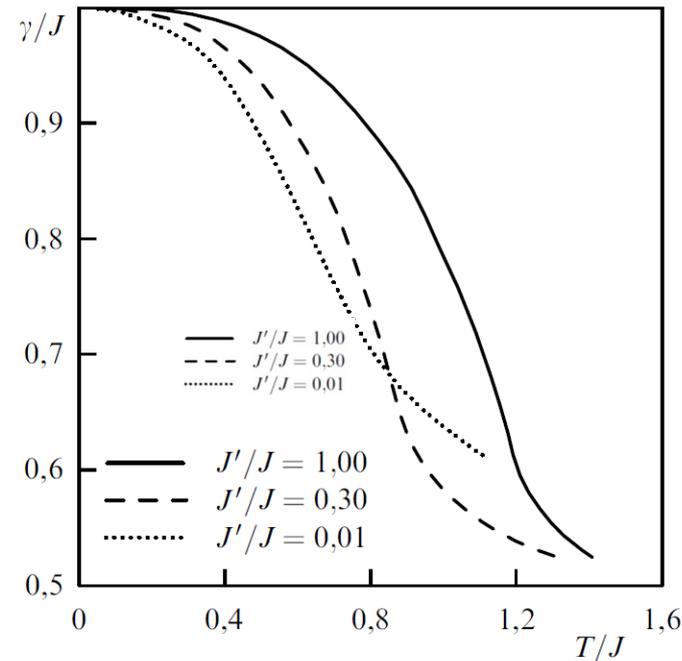
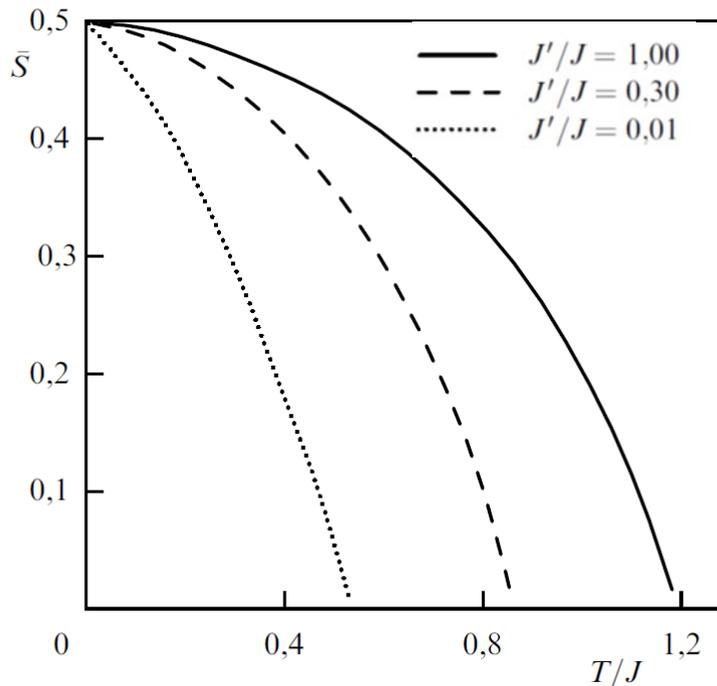
Alternative way: looks nicer but leads to some wrong numerical factors which should be eliminated by hands

SSWT of 2D magnetism III

Apart from magnetization – short-range order parameter describing magnon coupling on neighboring sites

$$\gamma = \bar{S} + \langle b_i^\dagger b_{i+\delta_\perp} \rangle, \quad \gamma' = \bar{S} + \langle b_i^\dagger b_{i+\delta_\parallel} \rangle$$

Layered 2D FM, no anisotropy, square-lattice in plane



SSWT of 2D magnetism IV

Logarithmic suppression of critical temperature; calculations beyond mean-field (ladder diagrams) are important

TABLE I. The experimental parameters and ordering temperatures of layered magnets and the corresponding calculated values of T_M in the standard spin-wave-theory (SWT), SSWT, and RPA (in brackets taking into account the constant $C_{AF} = -0.7$).

Compound	S	J (K)	J' (K)	Δ_0	T_M^{SWT} (K)	T_M^{SSWT} (K)	T_M^{RPA} (K)	T_M^{exp} (K)
La_2CuO_4	1/2	1600	0.8	≈ 0	672	537	343	325
K_2NiF_4	1	102	≈ 0	0.0088	160	125	90.0 (97.0)	97.1
Rb_2NiF_4	1	82	≈ 0	0.046	180	118	88.4 (95.0)	94.5
K_2MnF_4	5/2	8.4	≈ 0	0.015	74.8	52.1	42.7 (45.1)	42.1
CrBr_3	3/2	12.38	1.0	0.024	79.2	51.2	39.0	40.0

In the case of easy-plane anisotropy – no true long-range order, BKT transtion. But...

Dipole-dipole interactions in 2D magnetism

$$\mathcal{H}_{\text{dip}} = (g_e \mu_B)^2 \sum_{i < j} \left[\frac{(|\vec{r}_{ij}|^2 (\hat{S}_i \cdot \hat{S}_j) - 3(\hat{S}_i \cdot \vec{r}_{ij})(\hat{S}_j \cdot \vec{r}_{ij}))}{|\vec{r}_{ij}|^5} \right]$$

Long-range interaction formally **violating** conditions of Mermin-Wagner theorem. Tradition: to identify it with easy-plane shape-anisotropy. **But:** they are equivalent only for homogeneous magnetization and elliptic samples, statistical mechanics is **totally different!**

Maleev (1976): should be true long-range order since spin-wave spectrum is much harder: $\epsilon_k \sim k^{1/2}$

Confirmed by SSWT – will be important for discussions of CrCl_3

PHYSICAL REVIEW B **71**, 024427 (2005)

Thermodynamics of a two-dimensional Heisenberg ferromagnet with dipolar interaction

Alexei Grechnev,^{1,*} Valentin Yu. Irkhin,^{1,2} Mikhail I. Katsnelson,^{1,3} and Olle Eriksson¹

True long-range order at small T , first-order transition (?!)

The first attempt: K_2CuF_4

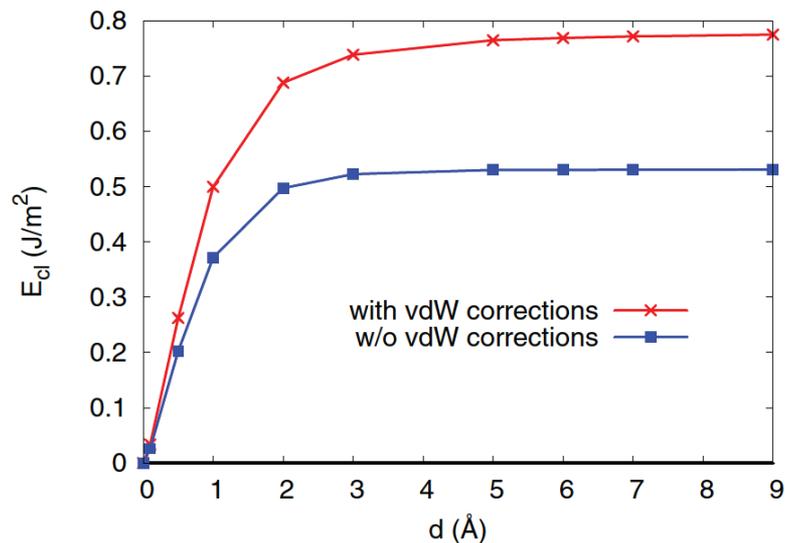
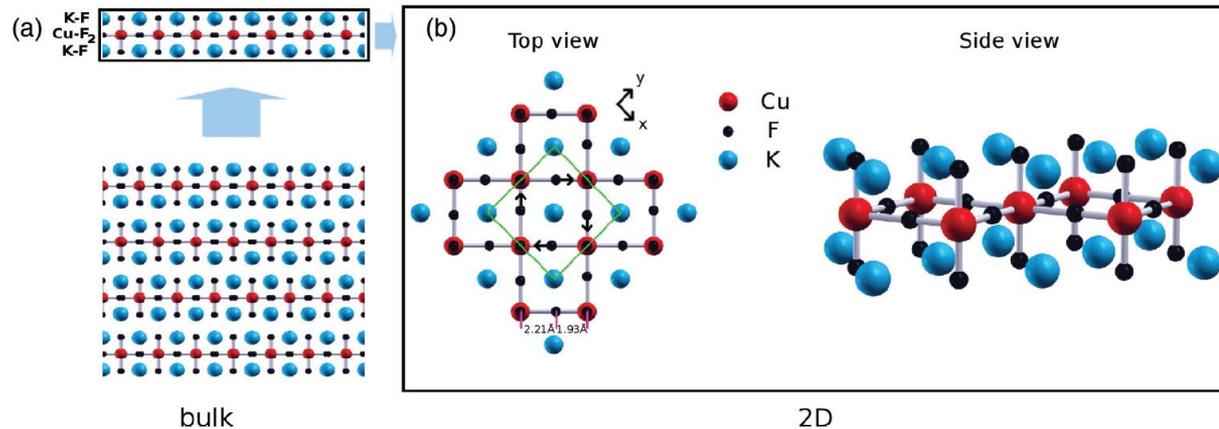
RAPID COMMUNICATIONS

PHYSICAL REVIEW B **88**, 201402(R) (2013)

Ferromagnetic two-dimensional crystals: Single layers of K_2CuF_4

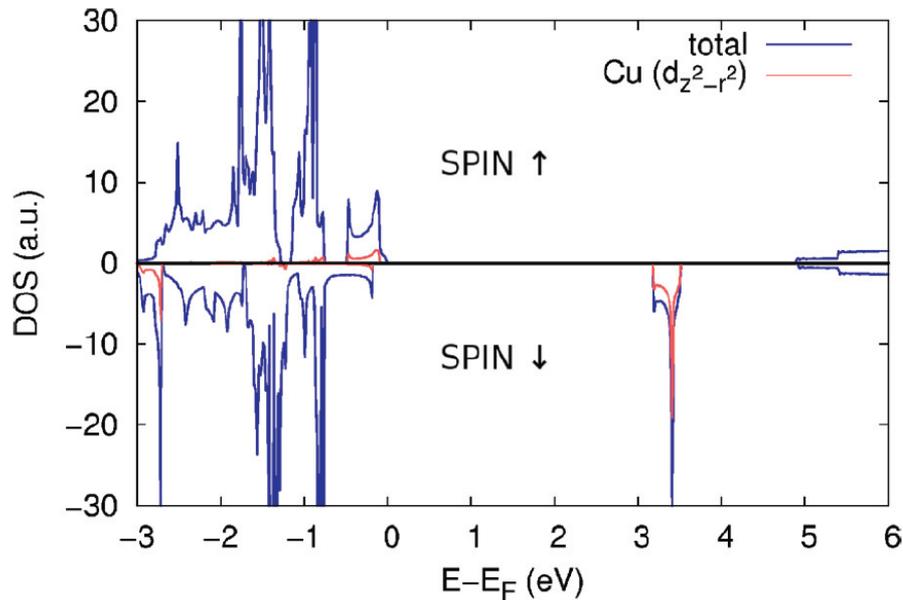
B. Sachs,^{1,*} T. O. Wehling,^{2,3} K. S. Novoselov,⁴ A. I. Lichtenstein,¹ and M. I. Katsnelson⁵

FM in layers, very weak AFM coupling between layers, very 2D – why not?



Cohesive energy is just 2-3 times larger than in graphite – exfoliation should be possible?!

The first attempt: K_2CuF_4 II



$$H = -\frac{1}{2} \sum_{\langle i,j \rangle} J [S_i^x S_j^x + S_i^y S_j^y + \eta S_i^z S_j^z]$$

LDA+U results for single layer

$$J/k_B = 25.3 \text{ K and } \eta = 0.90$$

Theoretical estimate of Kosterlitz-Thouless temperature about 8K.

May be still worth to try?! Small spin, strong quantum effects, BKT physics... Yes, temperatures are small but not too small

CrX₃: electronic structure

X=Cl, Br, I; probably, the most popular 2D magnets now, with reasonably high Curie temperatures

Correlation effects beyond DFT are important,
DFT+U is not sufficient

PHYSICAL REVIEW B **105**, 205124 (2022)

DMFT:

Dynamical correlations in single-layer CrI₃

Yaroslav O. Kvashnin,¹ Alexander N. Rudenko^{2,*} Patrik Thunström¹ Malte Rösner² and Mikhail I. Katsnelson²

PHYSICAL REVIEW B **104**, 155109 (2021)

Electronic structure of chromium trihalides beyond density functional theory

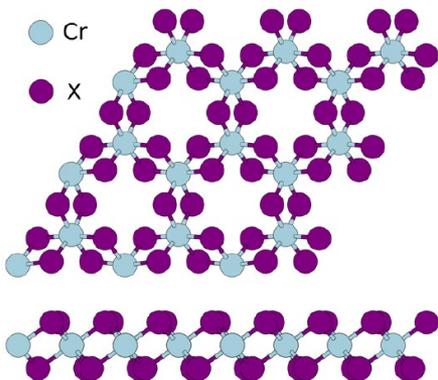
Swagata Acharya^{1,*} Dimitar Pashov,² Brian Cunningham³, Alexander N. Rudenko¹, Malte Rösner¹, Myrta Grüning,⁴ Mark van Schilfgaarde,^{2,5} and Mikhail I. Katsnelson¹

GW+BSE:

Importance of charge self-consistency in first-principles description of strongly correlated systems

Swagata Acharya¹✉, Dimitar Pashov², Alexander N. Rudenko¹, Malte Rösner¹, Mark van Schilfgaarde^{2,3} and Mikhail I. Katsnelson¹

npj Computational Materials (2021)7:208

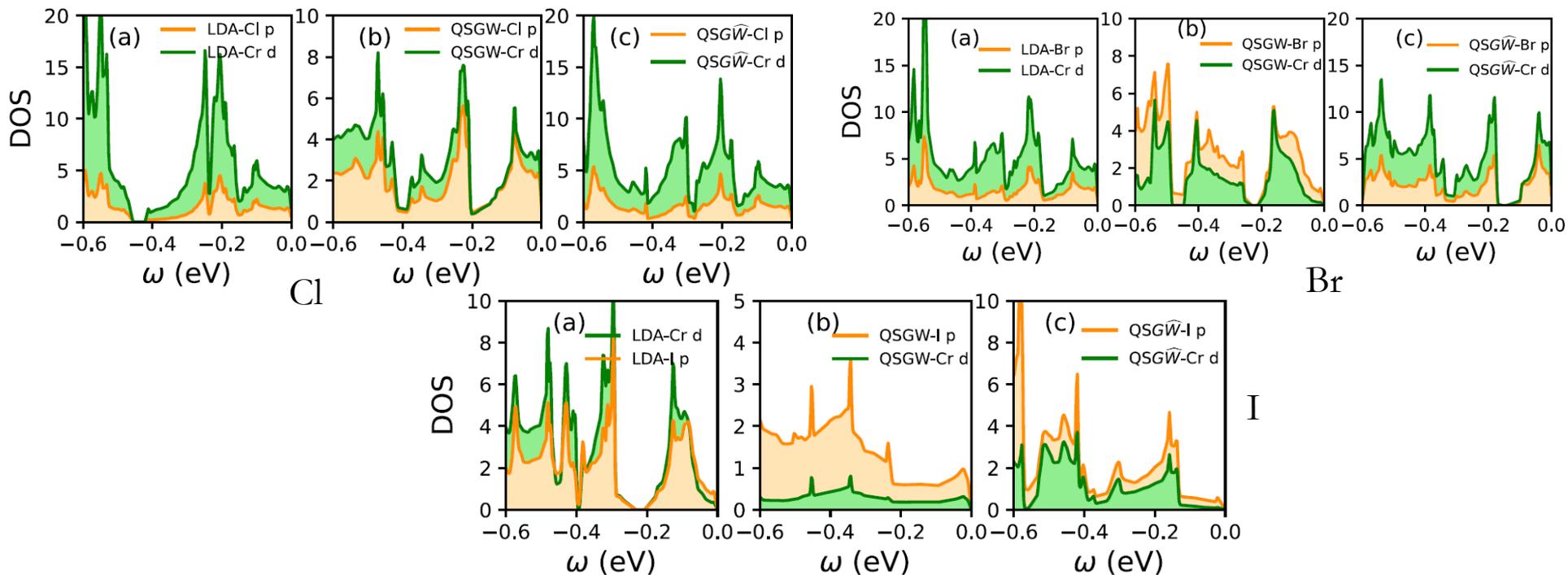


CrX₃: electronic structure II

Here we will focus on QSGW (+ BSE) results. The key effect is the contribution of Cr 3d states and ligand p-states into the valence band – different methods give different results; and of course the gap value!

Spectral weight of halogens in window (- 6 eV, 0 eV); ML – monolayer

Theory	ML band gap (eV)			ML spectral weight			Bulk band gap (eV)		
	CrCl ₃	CrBr ₃	CrI ₃	CrCl ₃	CrBr ₃	CrI ₃	CrCl ₃	CrBr ₃	CrI ₃
LDA	1.51	1.30	1.06	21%	26%	42%	1.38	1.2	0.91
QSGW	6.87	5.73	3.25	40%	63%	81%	5.4	4.38	3.0
QSGW \hat{W}	5.55	4.65	2.9	24%	31%	64%	4.4	3.5	2.5

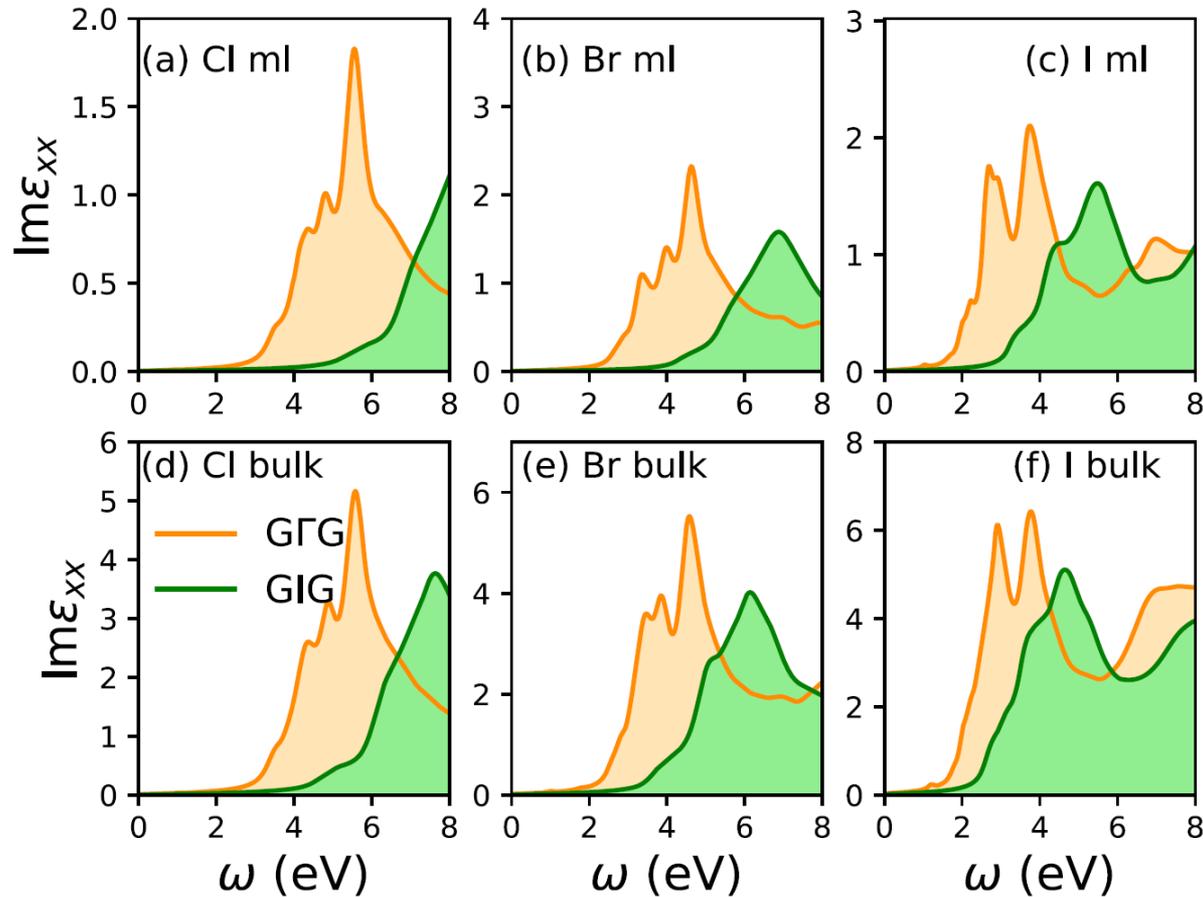


CrX₃: optics and excitons

Real- and momentum-space description of the excitons in bulk
and monolayer chromium tri-halides

npj 2D Materials and Applications (2022)6:33

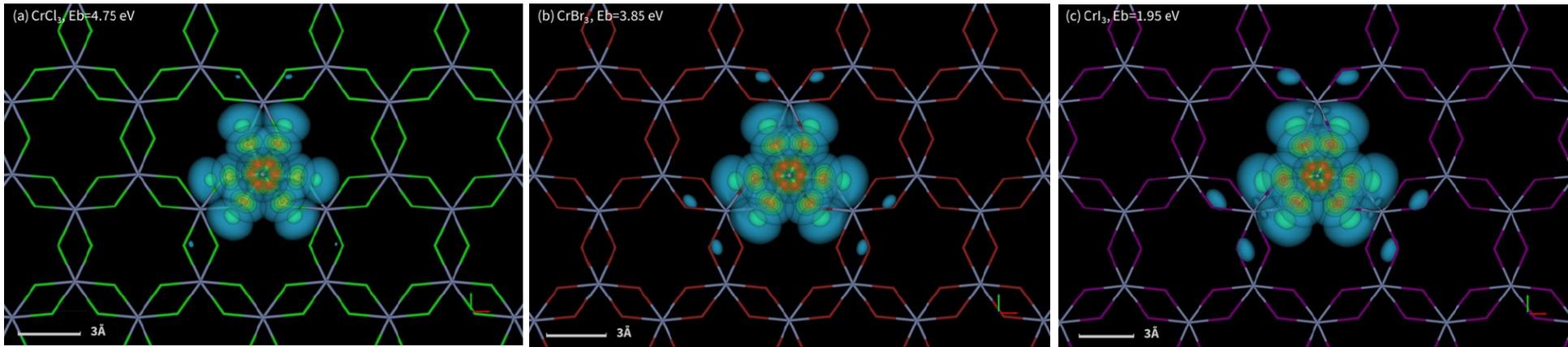
Swagata Acharya¹, Dimitar Pashov², Alexander N. Rudenko¹, Malte Rösner¹, Mark van Schilfgaarde^{2,3} and Mikhail I. Katsnelson¹



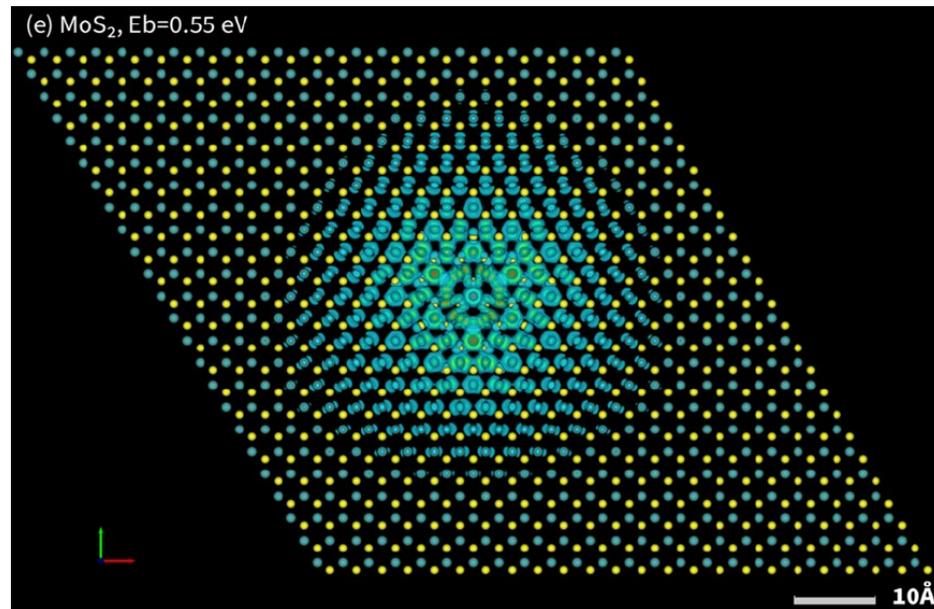
Orange – with vertex (that is, with excitons), green – just G^*G

CrX₃: optics and excitons II

When we increase d-p hybridization in the consequence Cl-Br-I, the deepest excitons become more delocalized but they are always close to Frenkel (local), limit than to Wannier-Mott limit



To compare: true Wannier-Mott excitons in MoS₂



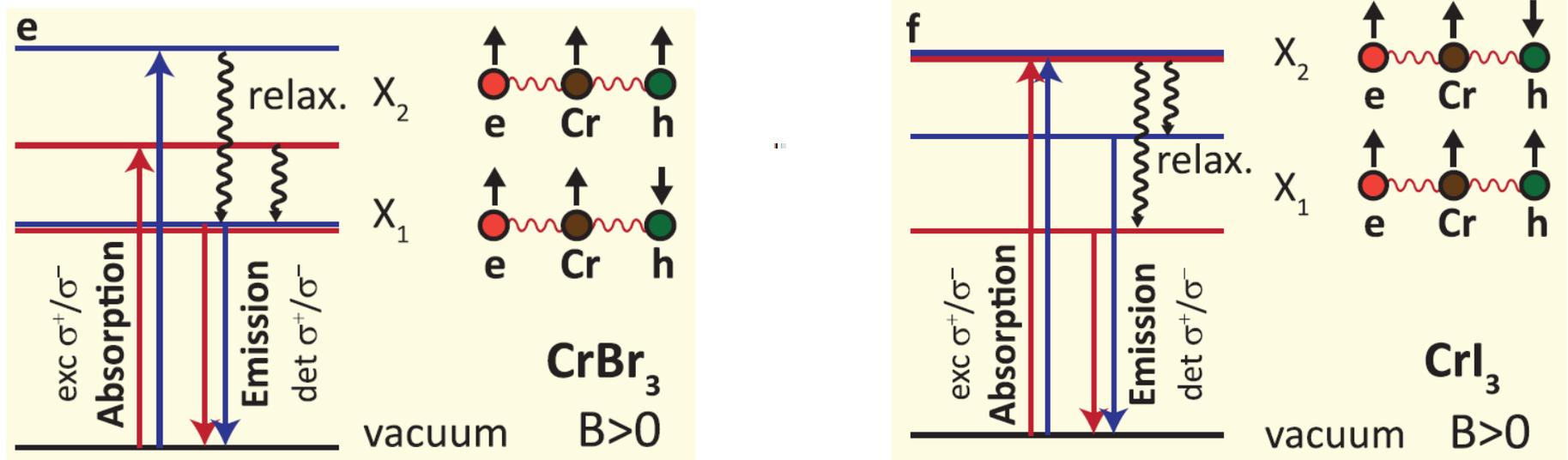
CrX₃: optics and excitons: Experiment

Strongly Correlated Exciton-Magnetization System for
Optical Spin Pumping in CrBr₃ and CrI₃.

Adv. Mater. 2023, 35, 2209513

M. Grzeszczyk, S. Acharya, D. Pashov, Z. Chen, K. Vaklinova, M. van Schilfgaarde,
K. Watanabe, T. Taniguchi, K. S. Novoselov, M. I. Katsnelson, and M. Koperski*

Luminescence in magnetic field shows opposite signs of effective e-h
exchange coupling in excitons in CrBr₃ and CrI₃



	Ground excitonic state		Excited excitonic state	
	Cr-electron	Cr-hole	Cr-electron	Cr-hole
CrBr ₃	FM	AFM	FM	FM
CrI ₃	FM	FM (small exchange constant)	FM	AFM

CrX₃: optics and excitons: Interpretation

Different role of d-d exchange with Cr atom (always FM, Hund exchange) and d-p exchange between Cr and halogen (always AFM, via Schrieffer-Wolf transformation). Small differences in the composition:

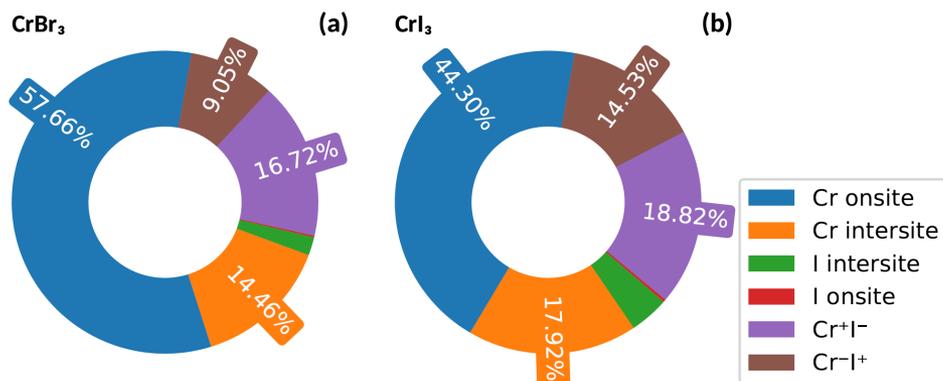


FIGURE S14 Frenkel and charge transfer components for first excited exciton (ex1) state in CrBr₃ (a) at 2.0 eV and in CrI₃ (b) at 1.9 eV.

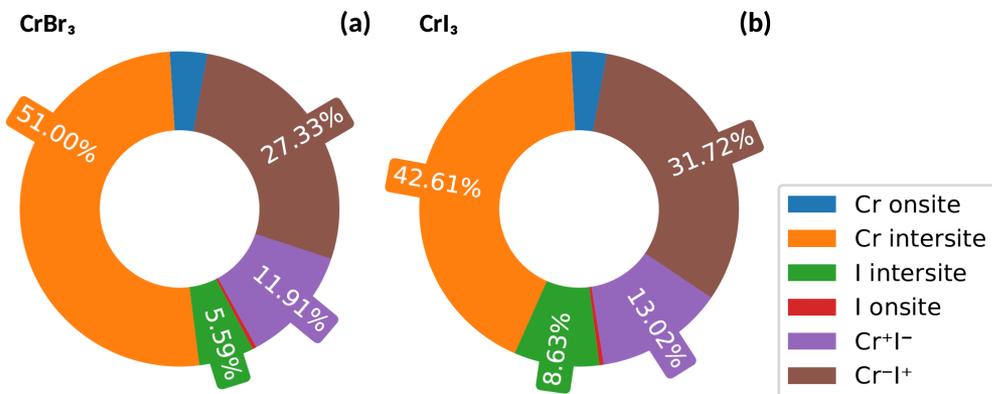


FIGURE S17 Frenkel and charge transfer components for second excited exciton (ex2) state in CrBr₃ (a) at 2.8 eV and in CrI₃ (b) at 2.2 eV.

Qualitatively: effective d-p exchange is FM when look at the whole valence band and AFM when look at its top; contributions of different energy regions are also different

Magnetic interactions in CrX_3

Correlation effects are important: quantum chemistry instead of DFT-based

Electronic excitations and spin interactions in chromium trihalides from embedded many-body wavefunctions

npj 2D Materials and Applications | (2024)8:56

Ravi Yadav ^{1,2}, Lei Xu ^{3,7}, Michele Pizzochero ⁴, Jeroen van den Brink ^{3,5}, Mikhail I. Katsnelson ⁶ & Oleg V. Yazyev ^{1,2} ✉

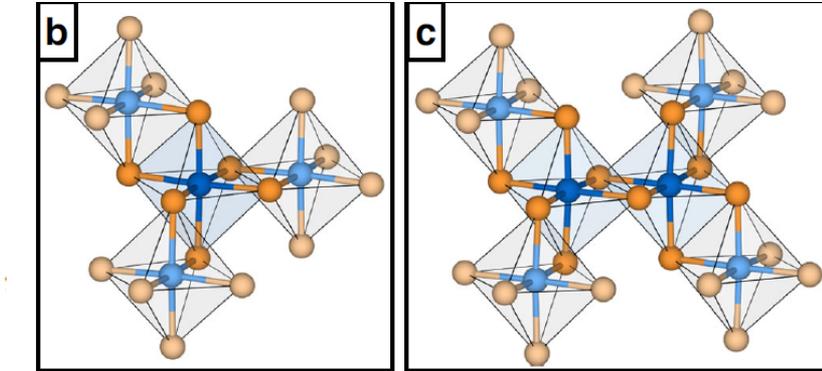
Embedding models used in calculations of intra-site and inter-site magnetic interactions

$$\mathcal{H} = \mathcal{H}^1 + \mathcal{H}^2$$

$$\mathcal{H}^1 = \sum_i \vec{S}_i \cdot \bar{D}_{\text{sia}} \cdot \vec{S}_i + \sum_i \mu_B \vec{B} \cdot \bar{g} \cdot \vec{S}_i$$

$$\mathcal{H}^2 = J_1 \sum_{i<j} (\vec{S}_i \cdot \vec{S}_j) + J_2 \sum_{i<j} (\vec{S}_i \cdot \vec{S}_j)^2 + \sum_{i<j} \vec{S}_i \cdot \bar{\Gamma} \cdot \vec{S}_j$$

$$\bar{\Gamma} = \begin{pmatrix} 0 & \Gamma_{xy} & -\Gamma_{yz} \\ \Gamma_{xy} & 0 & \Gamma_{yz} \\ -\Gamma_{yz} & \Gamma_{yz} & K \end{pmatrix}$$



K is the Kitaev interaction parameter

Magnetic interactions in CrX₃ II

Table 4 | Inter-site magnetic interactions in CrX₃ (X = Cl, Br, I), i.e., bilinear (J_1) and biquadratic (J_2) exchange couplings, along with the off-diagonal (Γ_{xy} , Γ_{yz} , Γ_{zx}) and Kitaev (K) parameters comprised in the symmetric anisotropic tensors, as obtained from the CASSCF and MRCI calculations

	CrCl ₃		CrBr ₃		CrI ₃	
	CASSCF	MRCI	CASSCF	MRCI	CASSCF	MRCI
J_1 (meV)	-0.64	-0.97	-0.61	-1.21	-0.60	-1.38
J_2 (meV)	-0.04	-0.05	-0.05	-0.05	-0.06	-0.06
Γ_{xy} (meV)	-1.2×10^{-4}	-2.1×10^{-4}	-6.9×10^{-4}	-0.8×10^{-3}	-1.2×10^{-3}	-4.2×10^{-4}
$\Gamma_{yz} = -\Gamma_{zx}$ (meV)	-1.3×10^{-5}	-0.8×10^{-4}	-6.9×10^{-4}	-0.9×10^{-3}	-1.1×10^{-4}	-3.1×10^{-4}
K (meV)	-1.7×10^{-4}	-1.1×10^{-4}	-8.2×10^{-3}	-0.01	-9.3×10^{-3}	-0.05

Biquadratic exchange is weak, Kitaev interaction is extremely weak

Table 3 | Comparison of the dipolar anisotropy parameter A_{dip} and the single-ion anisotropy parameter A_{sia} obtained from MRCI calculations for bulk CrX₃ (X = Cl, Br, I)

	CrCl ₃	CrBr ₃	CrI ₃
A_{sia} (μeV)	-31.4	-81.1	-124.2
A_{dip} (μeV)	85.2	75.5	57.3

The negative sign indicates that the out-of-plane spin moments are favored. The shape anisotropy parameter A_{dip} for monolayer CrX₃ (X = Cl, Br, I) approximately doubles in magnitude as compared to the bulk, and attains values of 144 μeV , 121 μeV , 94 μeV for chloride, bromide and iodide, respectively.

For X = I, magnetocrystalline anisotropy wins for sure, easy-axis magnetism. For X = Cl, dipole-dipole wins, in-plane magnetization but true long-range order rather than BKT. For X = Br, they are comparable.

Magnetic polaron in bilayer CrI_3

General statement: small doping of magnetic insulators always favor FM since it is more favorable for one electron (or hole) move in FM environment (Zener double exchange, Nagaoka ferromagnetism...)

Idea: if concentration of electrons is not sufficient to establish FM order in the whole crystal, it can be established locally: charge carriers are self-trapped in FM droplet

Ferron (Nagaev), fluctuon (Krivoglaz), magnetic polaron (Mott and many others)...

Formal theory based on path integral formalism: Auslender & MIK 1980-1982

Example: effective spin Hamiltonian for narrow-band Hubbard (or s-d exchange) model at Bethe lattice (Auslender & MIK 1982)

$$\Phi(\{\mathbf{n}_i\}) \approx \text{const} + \frac{zt^2}{2\epsilon_0} \sum_i \cos \theta_i - \left(\frac{8}{15\pi z^{1/2}}\right) z|t| \sum_i \cos\left(\frac{\theta_i}{2}\right) \left(\frac{\bar{\mu} - E_{it}}{z^{1/2}|t|\cos(\theta_i/2)}\right)^{1/2} \Theta(\bar{\mu} - E_{it}); \quad \Theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases}$$

$$\cos \theta_i = z^{-1} \sum_{\delta} (\mathbf{n}_i \mathbf{n}_{i+\delta})$$

Favors phase separation!

Magnetic polaron in bilayer CrI₃ II

PHYSICAL REVIEW B **101**, 041402(R) (2020)

Rapid Communications

Editors' Suggestion

Magnetic polaron and antiferromagnetic-ferromagnetic transition in doped bilayer CrI₃

D. Soriano  and M. I. Katsnelson

Application to AFM bilayer CrI₃ (contrary to bulk, interlayer exchange is AFM)

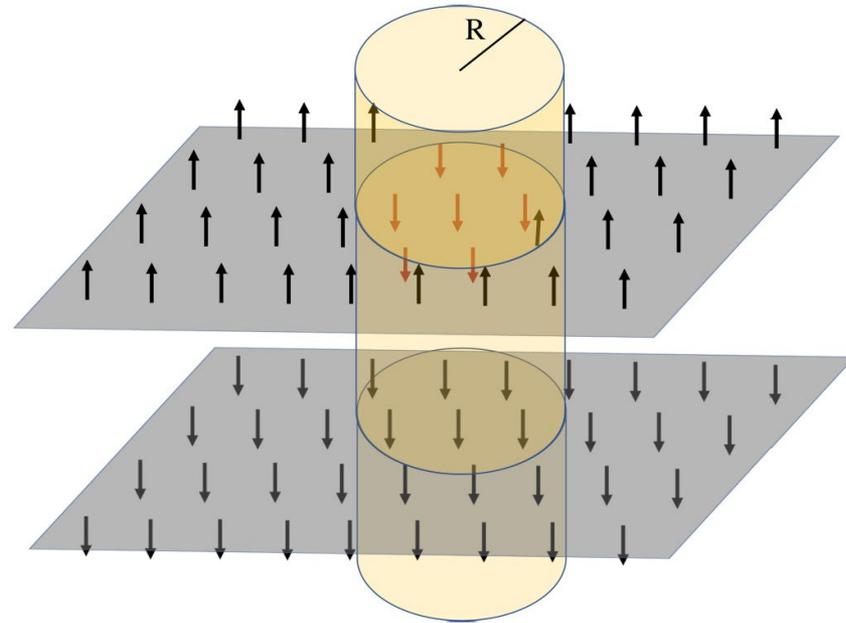
Simple estimate due to Mott (1974)

$$\mathcal{E}(R) = -\Delta + \frac{\hbar^2 z_0^2}{2m^* R^2} + J \frac{\pi R^2}{S_0}, \quad (2)$$

where the first term Δ is the carrier energy difference between parallel and antiparallel interlayer magnetization, namely, $\Delta_{e(h)} = |\epsilon_{\text{CB(VB)}}^{\text{AFM}} - \epsilon_{\text{CB(VB)}}^{\text{FM}}|$, where VB and CB stand for the top and bottom of valence and conduction bands, respectively. The second term is the energy of a particle confined in a disk of radius R (Fig. 1), with $z_0 = 2.40483$ being the first zero of the Bessel function $J_0(z)$, and m^* is the corresponding effective carrier mass in the ferromagnetic state. The third term is the exchange energy needed to switch the magnetic interaction between adjacent layers, where J is the interlayer exchange coupling per magnetic atom, $S_0 = 3a^2\sqrt{3}/4$ is the area occupied by a single Cr atom in the unit cell, and a is the Cr-Cr intralayer distance.

Optimal energy and radius:

$$W = \frac{\hbar^2}{2m^* a^2} \quad \frac{R^*}{a} = \left(\frac{W}{J} \frac{z_0^2 3\sqrt{3}}{4\pi} \right)^{1/4} \quad \mathcal{E} = -\Delta + 2\sqrt{WJ \frac{4\pi z_0^2}{3\sqrt{3}}}$$



Magnetic polaron in bilayer CrI₃ III

Simple DFT estimate:

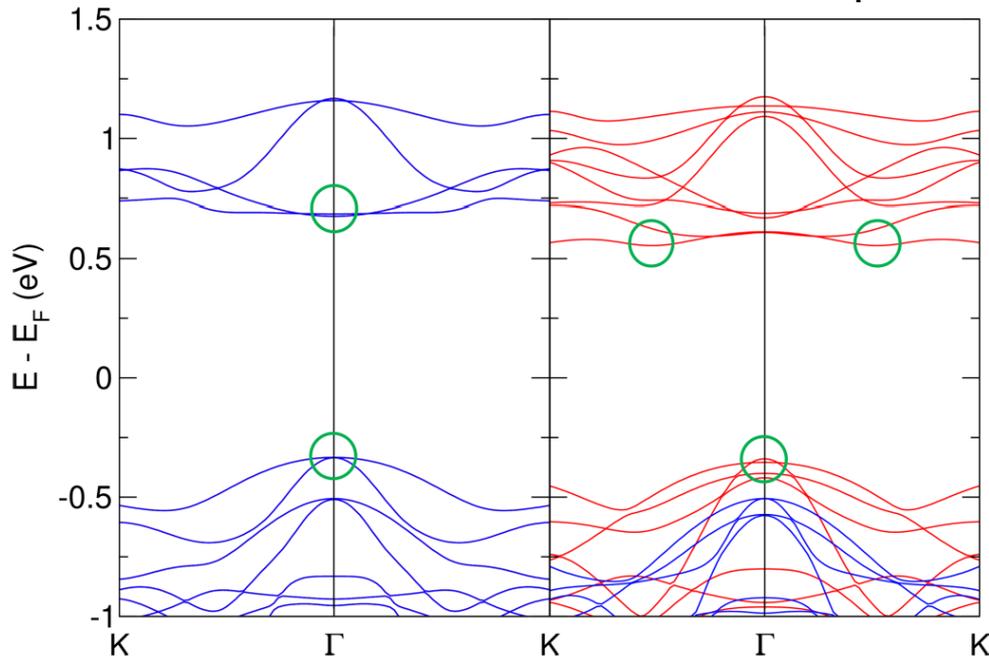


FIG. 3. Band structure of monoclinic bilayer CrI₃ for AFM (left) and FM (right) interlayer exchange. The green circles denote the maximum and minimum of the valence and conduction bands, respectively.

TABLE I. Summary of the parameters obtained from first principles (m_e is the free electron mass).

	VBMax (eV)	CBMin (eV)	m_h^*/m_e	m_e^*/m_e
AFM	-0.333	0.678	0.15	0.18
FM	-0.339	0.555	0.02	0.11

$$\Delta_e \approx 123 \text{ meV and } \Delta_h \approx 6 \text{ meV}$$

$$(R_e^*/a) = 28.4 \text{ and } (R_h^*/a) = 43.4$$

$$\mathcal{E}_e = -91.3 \text{ meV and } \mathcal{E}_h = 67.9 \text{ meV}$$

Prediction: for hole-doped case self-trapping is impossible but for electron-doped case it is, With self-localization energy about 100 meV

CrSBr: tunability of magnetic properties

Dielectric tunability of magnetic properties in orthorhombic ferromagnetic monolayer CrSBr

Alexander N. Rudenko¹✉, Malte Rösner¹ and Mikhail I. Katsnelson¹

npj Computational Materials (2023)9:83

Highly anisotropic crystal structure, FM in monolayer with Curie temperature 146 K

Correlations are very important, minimal method is DFT+U but U should be calculated ab initio

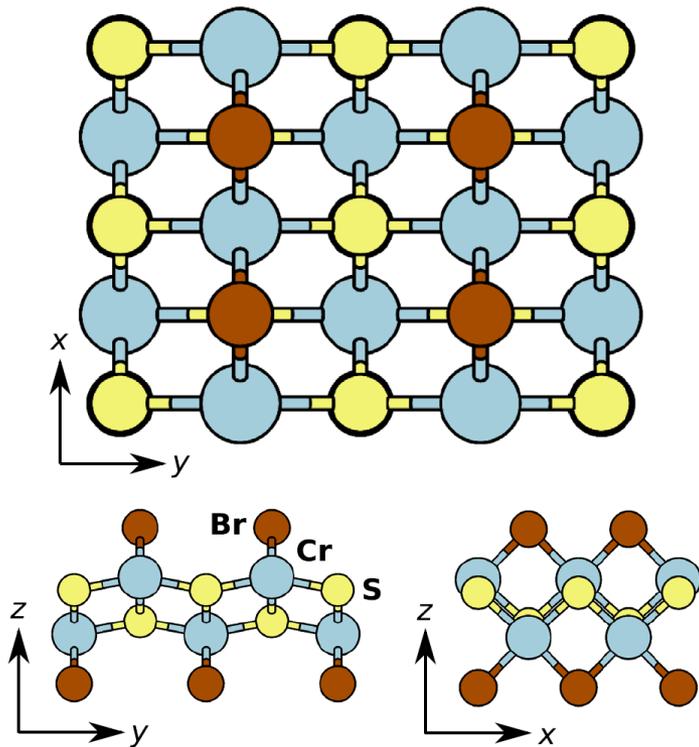


Fig. 1 Schematic crystal structure of monolayer CrSBr shown in three different projections. Brown, blue, and yellow balls correspond to Br, Cr, and S atoms, respectively.

Table 1. WFCE + cRPA averaged local Cr *d* Coulomb matrix elements as a function of the screening from a dielectric encapsulation of the CrSBr monolayer.

ϵ_{env}	U (eV)	U' (eV)	J_H (eV)	$U_{eff} = U - J_H$ (eV)
1	3.68	2.86	0.39	3.28
2	3.15	2.34	0.39	2.76
4	2.74	1.93	0.39	2.35
8	2.43	1.61	0.39	2.03
16	2.20	1.39	0.39	1.81
32	2.06	1.25	0.39	1.67
64	1.98	1.17	0.39	1.58
∞^a	1.90	1.08	0.39	1.40

CrSBr: tunability of magnetic properties II

Magnetic Hamiltonian (parameters from fitting to DFT+U total energies of different spin configurations)

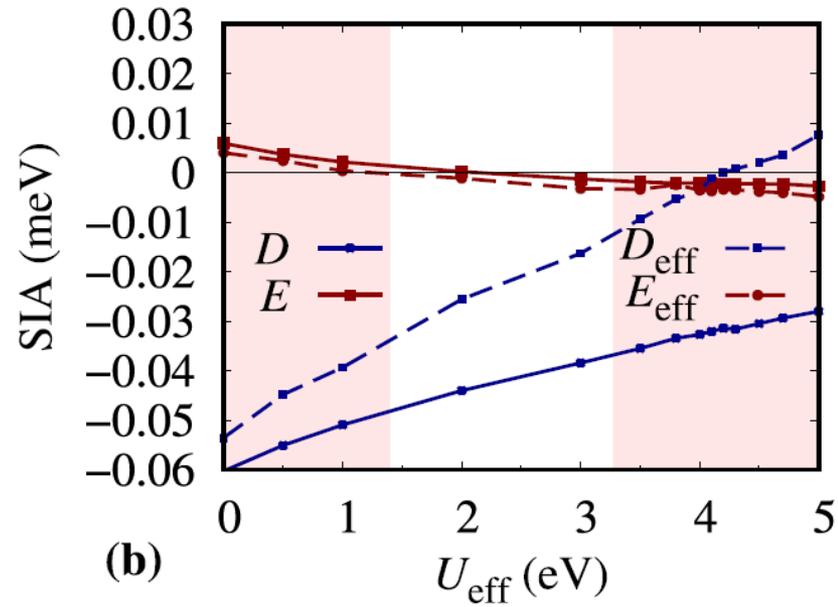
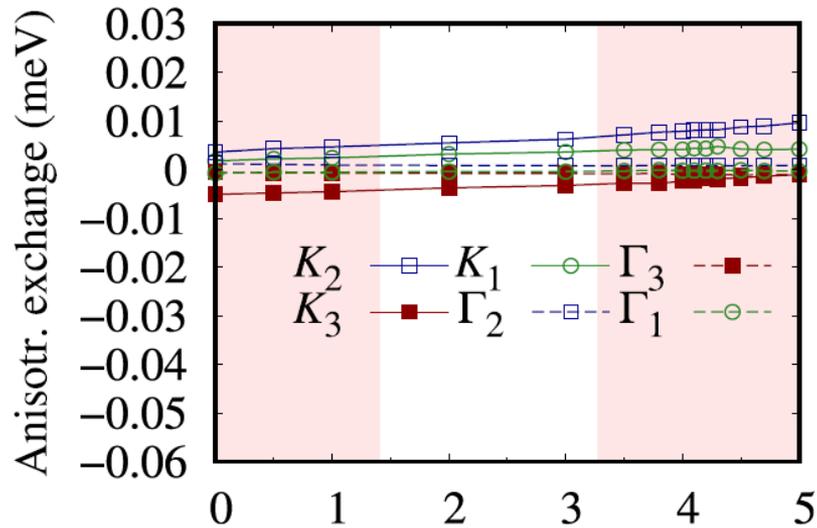
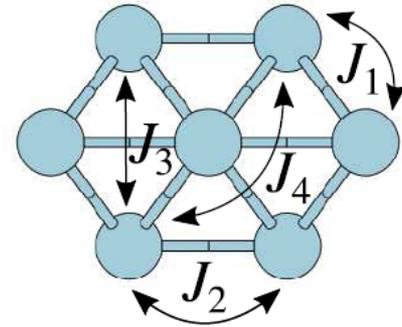
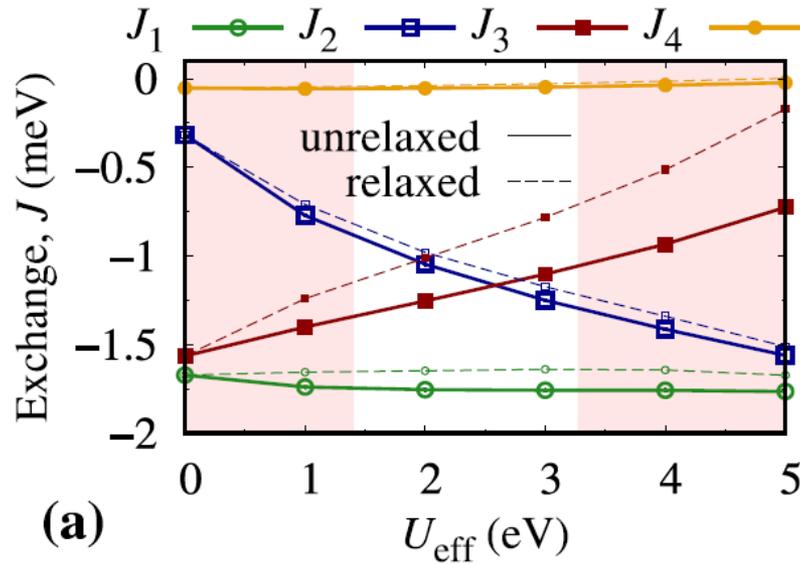
$$H = H_0 + H_{\text{SIA}} + H_{\text{AE}} + H_{\text{D}}$$

$$H_0 = \sum_{ij} J_{ij} \mathbf{S}_i \mathbf{S}_j \quad H_{\text{SIA}} = D \sum_i (S_i^y)^2 + E \sum_i \left[(S_i^z)^2 - (S_i^x)^2 \right]$$

$$H_{\text{AE}} = \sum_{ij} K_{ij} S_i^y S_j^y + \sum_{ij} \Gamma_{ij} (S_i^z S_j^z - S_i^x S_j^x)$$

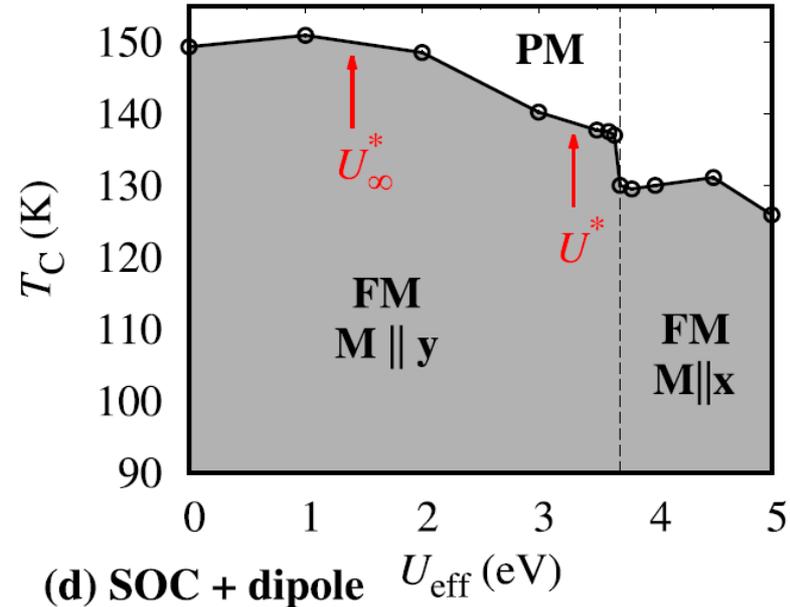
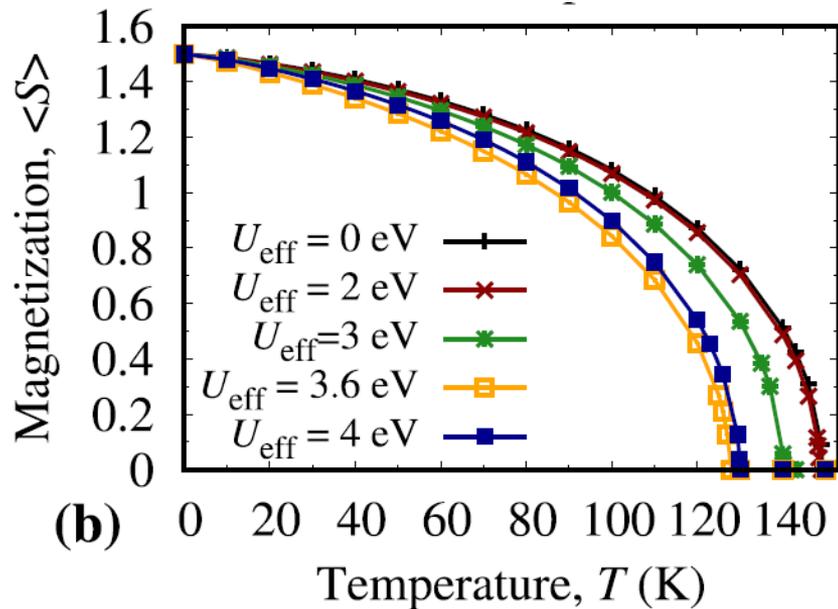
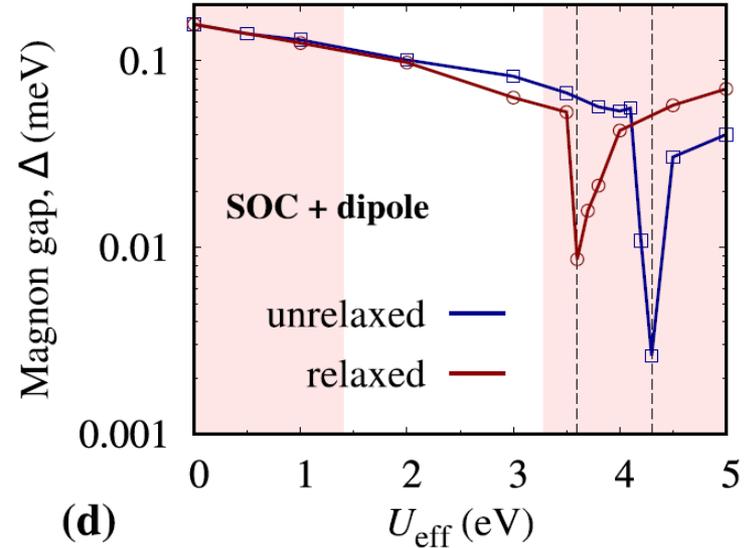
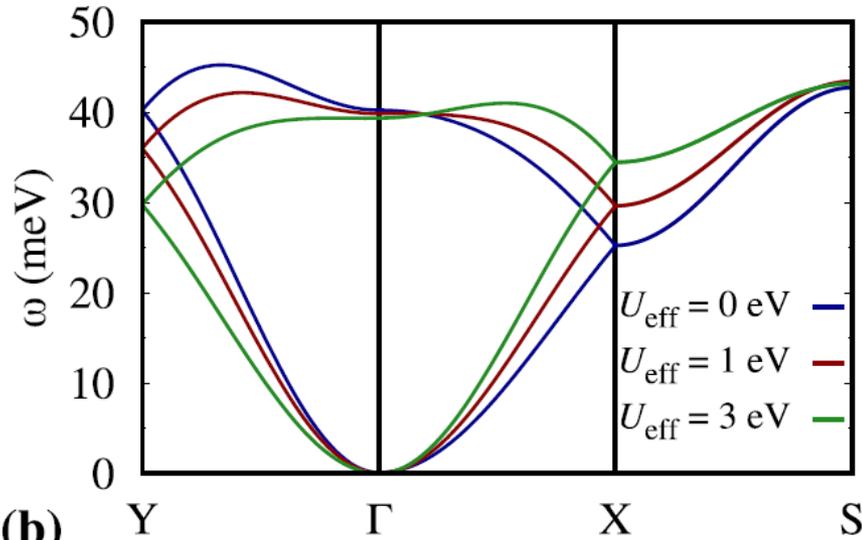
$$H_{\text{D}} = \frac{\Omega}{2} \sum_{ij} \frac{1}{|\mathbf{R}_{ij}|^3} \left(\mathbf{S}_i \mathbf{S}_j - 3 \frac{(\mathbf{S}_i \cdot \mathbf{R}_{ij})(\mathbf{S}_j \cdot \mathbf{R}_{ij})}{\mathbf{R}_{ij}^2} \right)$$

CrSBr: tunability of magnetic properties III



CrSBr: tunability of magnetic properties IV

relaxed



Hyperbolic polaritons in (bulk) CrSBr

Just to remind:
crystallooptics

$$\vec{k} = k_0 \vec{n}, \quad k_0 = \frac{\omega}{c}$$

$$\det \left| n^2 \delta_{ij} - n_i n_j - \varepsilon_{ij}(\omega) \right| = 0$$

$$\varepsilon_{ij}(\omega) = \delta_{ij} + \frac{4\pi i}{\omega} \sigma_{ij}(\omega)$$

Main axes $\varepsilon_{ij} = \varepsilon_i \delta_{ij}$

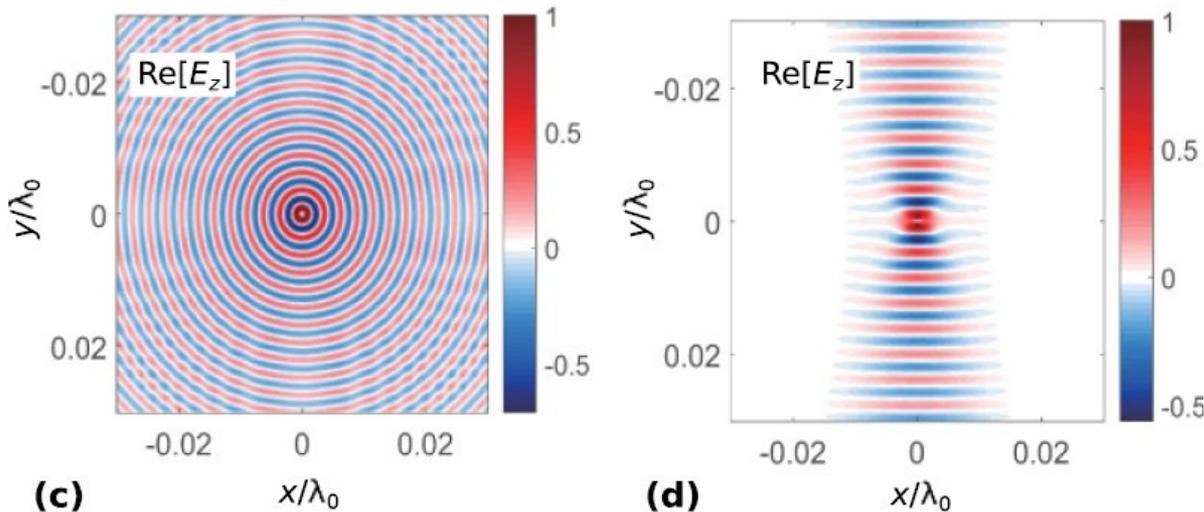
If $\varepsilon_x \varepsilon_y < 0$

$$\begin{aligned} n_z &= 0 \\ n^2 &= \varepsilon_z \\ \frac{n_x^2}{\varepsilon_y} + \frac{n_y^2}{\varepsilon_x} &= 1 \end{aligned}$$

$$\frac{k_x^2}{|\varepsilon_y|} = \frac{k_y^2}{|\varepsilon_x|}, \quad k \gg k_0 \quad (c \rightarrow \infty)$$

Anisotropy can result in the formation of additional types of electromagnetic waves!

Hyperbolic polaritons in bulk CrSBr II



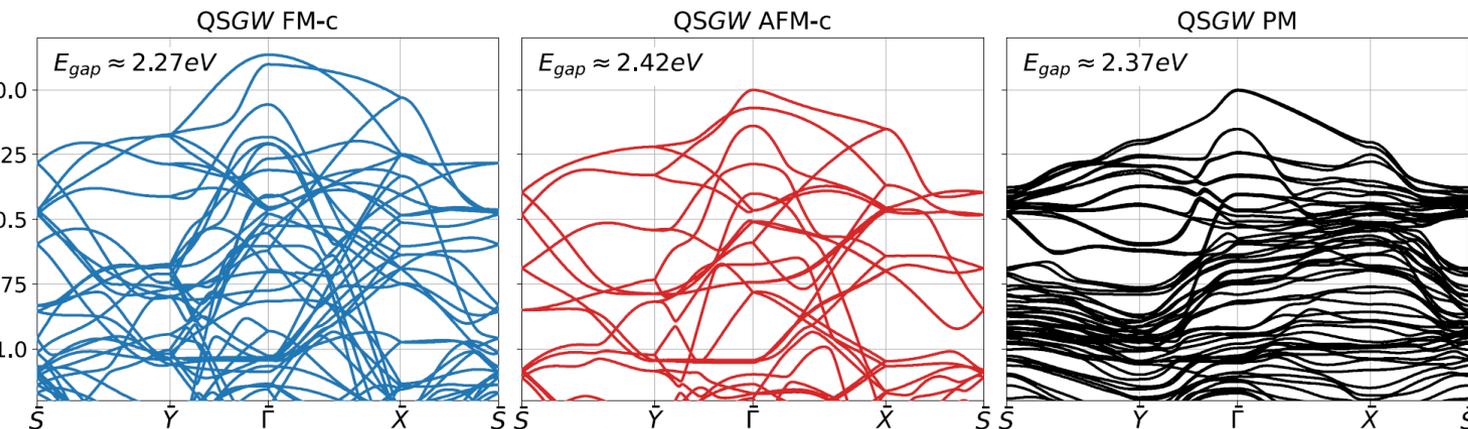
Distribution of electric fields for elliptic and hyperbolic polaritons

Chang P-H, Lin C and Helmy A S 2022 Field canalization using anisotropic 2D plasmonics *npj 2D Mater. Appl.* **6** 5

In CrSBr: electronic structure and thus optical properties are sensitive to magnetic order

Paramagnetic electronic structure of CrSBr: Comparison between *ab initio* GW theory and angle-resolved photoemission spectroscopy

Marco Bianchi ¹, Swagata Acharya ^{2,3}, Florian Dirnberger ⁴, Julian Klein ⁵, Dimitar Pashov ⁶, Kseniia Mosina ⁷, PHYSICAL REVIEW B **107**, 235107 (2023)
 Zdenek Sofer ⁷, Alexander N. Rudenko ³, Mikhail I. Katsnelson ³, Mark van Schilfgaarde ^{6,2}
 Malte Rösner ³ and Philip Hofmann ^{1,*}



Valence bands in QSGW

Hyperbolic polaritons in bulk CrSBr III

nature communications



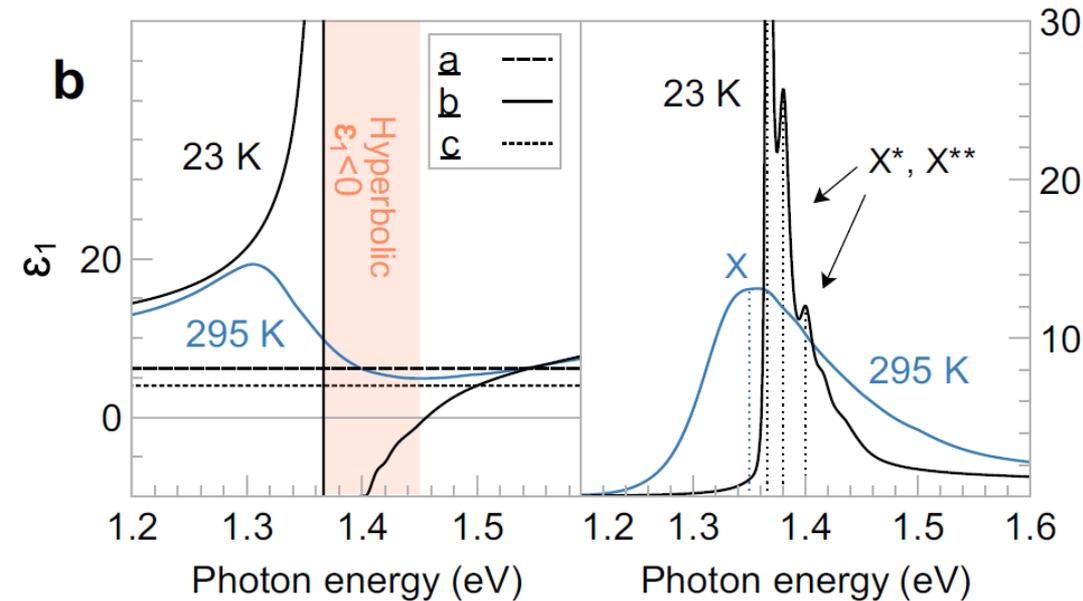
Francesco L. Ruta^{1,2,12}✉, Shuai Zhang^{1,12}✉, Yinming Shao^{1,12}, Samuel L. Moore¹, Swagata Acharya³, Zhiyuan Sun¹, Siyuan Qiu¹, Johannes Geurs^{1,4}, Brian S. Y. Kim^{1,5}, Matthew Fu¹, Daniel G. Chica⁶, Dimitar Pashov⁷, Xiaodong Xu^{8,9}, Di Xiao^{8,9}, Milan Delor⁶, X-Y. Zhu⁶, Andrew J. Millis^{1,10}, Xavier Roy⁶, James C. Hone⁵, Cory R. Dean¹, Mikhail I. Katsnelson¹¹, Mark van Schilfgaarde³ & D. N. Basov¹✉

Article

<https://doi.org/10.1038/s41467-023-44100-6>

Hyperbolic exciton polaritons in a van der Waals magnet

Experiment: observation of hyperbolic polariton at low enough temperature

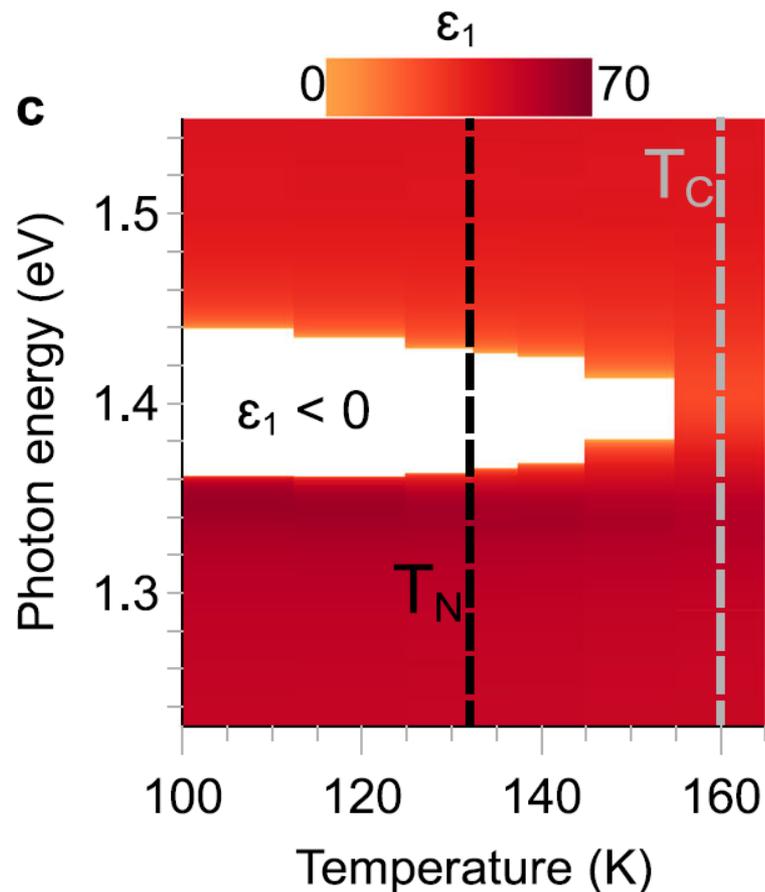


Resonances associated to excitons
(confirmed by QSGW+BSE
calculations)

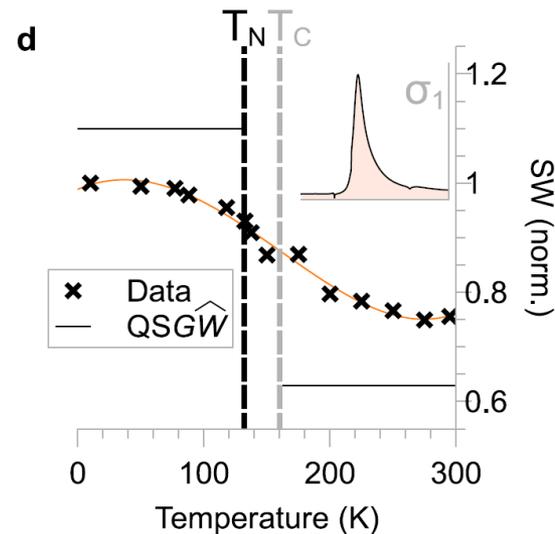
AFM order is crucially important!

Hyperbolic polaritons in bulk CrSBr IV

Hyperbolic region from experiment



Exciton spectral weight
(experiment, cf two QSGW
values for AFM and PM states)



Antiferromagnetic



Paramagnetic



Cr intersite Cr onsite $\text{Cr}^+ \text{Br}^- \text{S}^2 \text{S}^2$

Exciton decomposition (calculations)

Lattice effects and magnetism in Fe_3GeTe_2

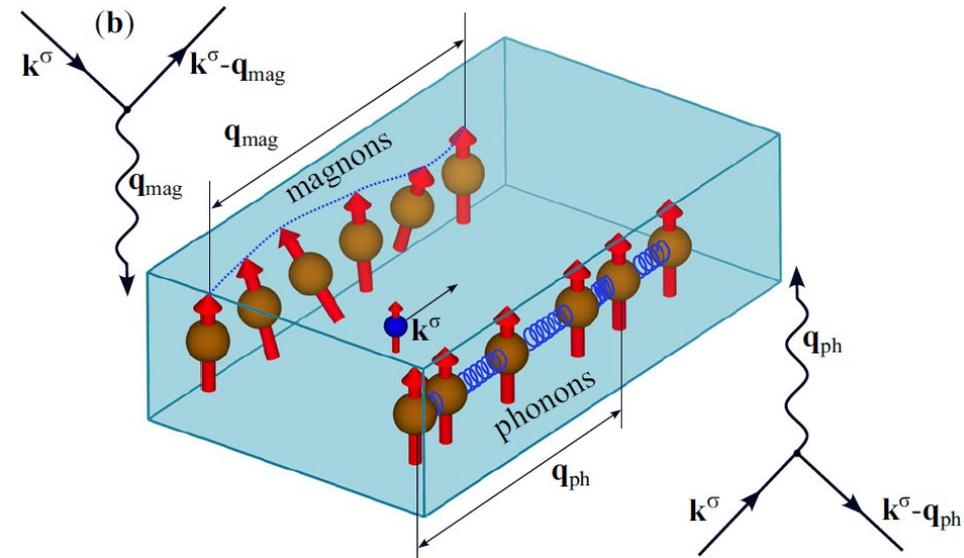
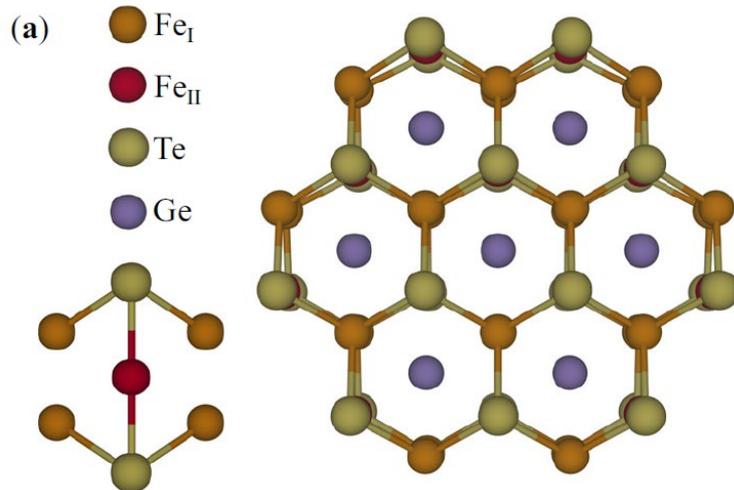
Electron transport and scattering mechanisms in ferromagnetic monolayer Fe_3GeTe_2

npj 2D Materials and Applications (2023)7:52

Danis I. Badrtdinov¹✉, Georgy V. Pushkarev², Mikhail I. Katsnelson¹ and Alexander N. Rudenko¹✉

Metallic 2D FM with high Curie temperature
(above 200 K)

Comparison of magnon
and phonon scattering
processes



Lattice effects and magnetism in Fe_3GeTe_2 II

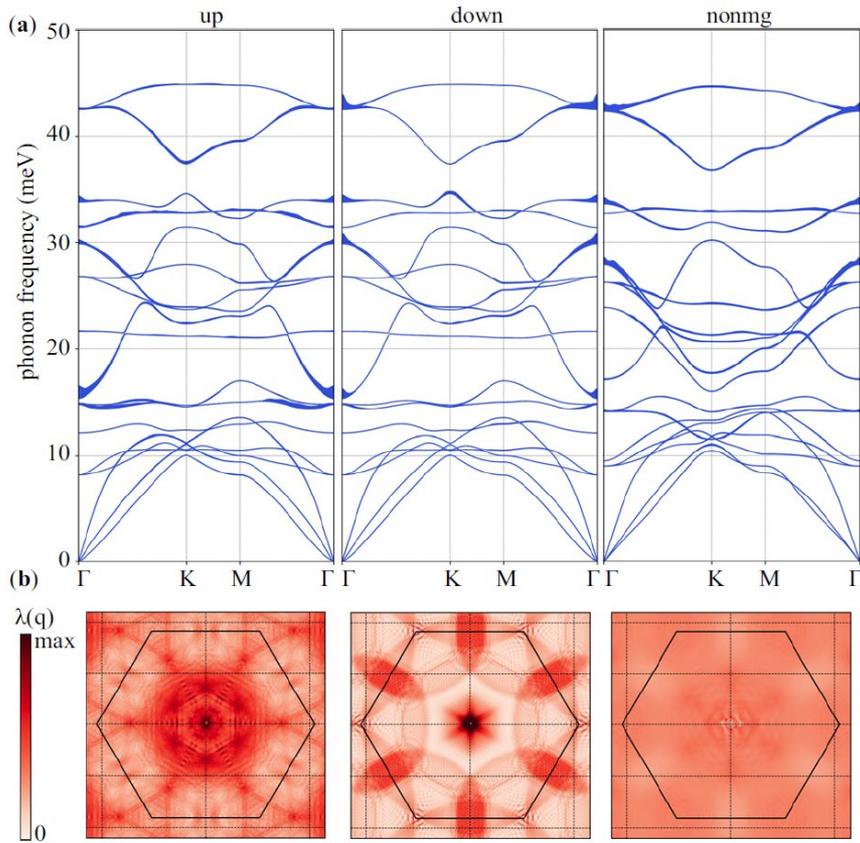


Fig. 3 Phonon dispersion curves and momentum-resolved electron-phonon coupling calculated for the ferromagnetic and nonmagnetic phases of monolayer Fe_3GeTe_2 . **a** Phonon dispersion curves shown with the spin-dependent linewidth that is proportional to $\text{Im}[\Pi_{\mathbf{q}\nu}^\sigma] = \pi N_F^e \lambda_{\mathbf{q}\nu}^\sigma \omega_{\mathbf{q}\nu}^2$. **b** Momentum-resolved electron-phonon coupling constant $\lambda_{\mathbf{q}}^\sigma = \sum_\nu \lambda_{\mathbf{q}\nu}^\sigma$.

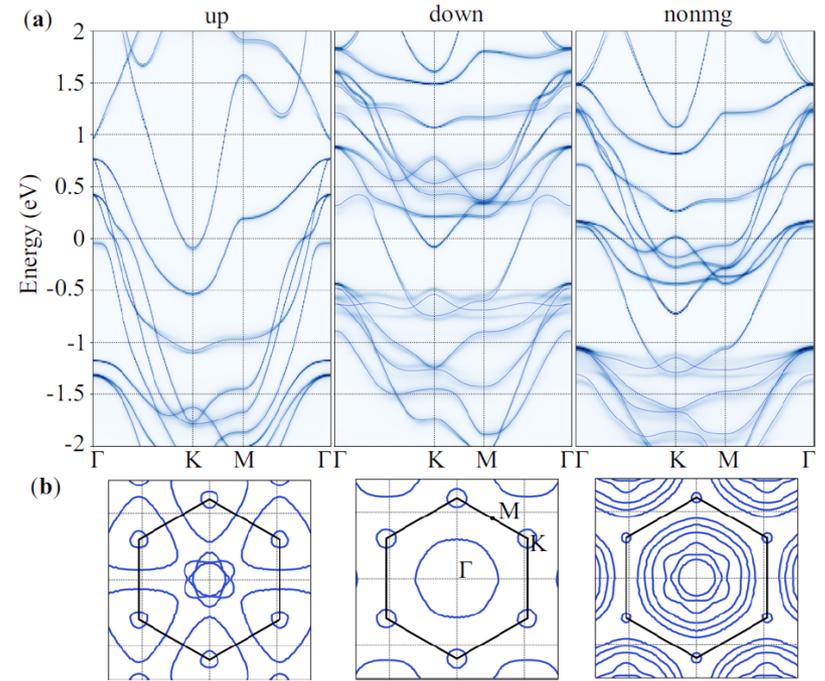
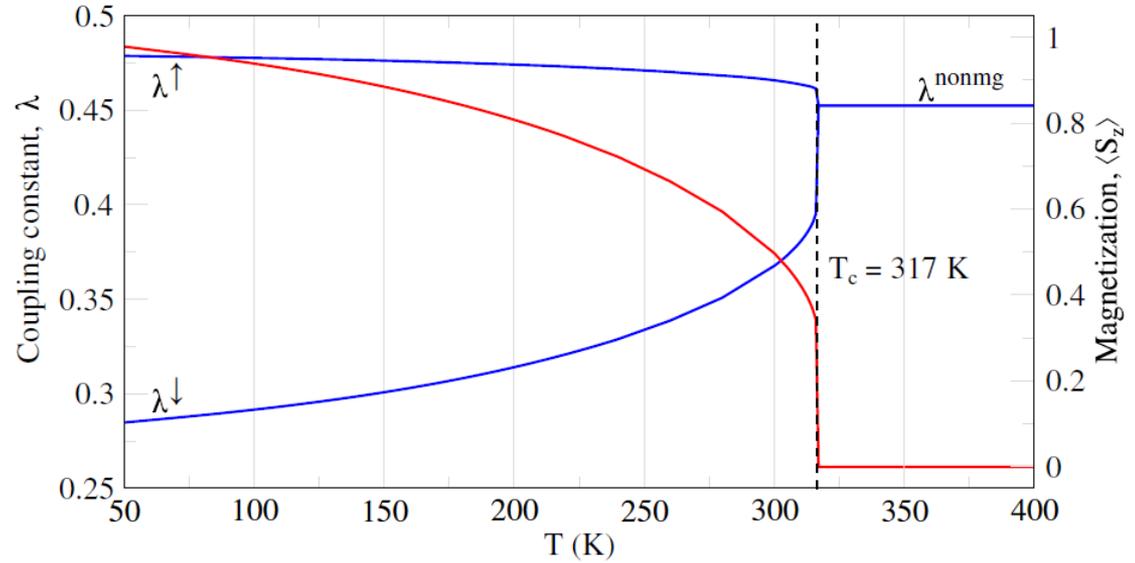


Fig. 2 Spin-dependent electron spectral functions and Fermi contour maps for the ferromagnetic and nonmagnetic phases of monolayer Fe_3GeTe_2 . **a** Spectral functions $A_{\mathbf{k}}^\sigma(\omega, T) = -1/\pi \text{Im}[G_{\mathbf{k}}^\sigma(\omega, T)]$ calculated in the presence of the electron-phonon interactions for $T=100$ K for the states near the Fermi level. Original DFT band structure is shown by the blue solid line. Zero energy corresponds to the Fermi energy. **b** The corresponding Fermi contour maps.

Lattice effects and magnetism in Fe₃GeTe₂ III

Effective electron-phonon coupling constant by simple interpolation

$$\lambda^\sigma(T) = \lambda_{\text{nonmg}} + [\lambda^\sigma - \lambda_{\text{nonmg}}] \frac{\langle S_z \rangle}{S}$$



Magnetization from exchange parameters by magnetic force theorem plus generalized Tyablikov approximation

$$J_{ij} = \frac{1}{2\pi S^2} \int_{-\infty}^{E_F} d\varepsilon \text{Im} \left[\Delta_i G_{ij}^\downarrow(\varepsilon) \Delta_j G_{ji}^\uparrow(\varepsilon) \right]$$

$$\hat{\mathcal{H}}_{\mu\nu}^{SW}(\mathbf{q}) = \left[\delta_{\mu\nu} \left[A + \sum_{\chi} J_{\mu\chi}(\mathbf{0}) \right] - J_{\mu\nu}(\mathbf{q}) \right] \langle S_z \rangle$$

$$\langle S_z \rangle = S \frac{1 + 2 \sum_{\mathbf{q}\nu} b_{\mathbf{q}\nu}}{1 + 3 \sum_{\mathbf{q}\nu} b_{\mathbf{q}\nu} + 3 (\sum_{\mathbf{q}\nu} b_{\mathbf{q}\nu})^2}$$

$$b_{\mathbf{q}\nu} = (\exp[\hbar\omega_{\mathbf{q}\nu}/k_B T] - 1)^{-1}$$

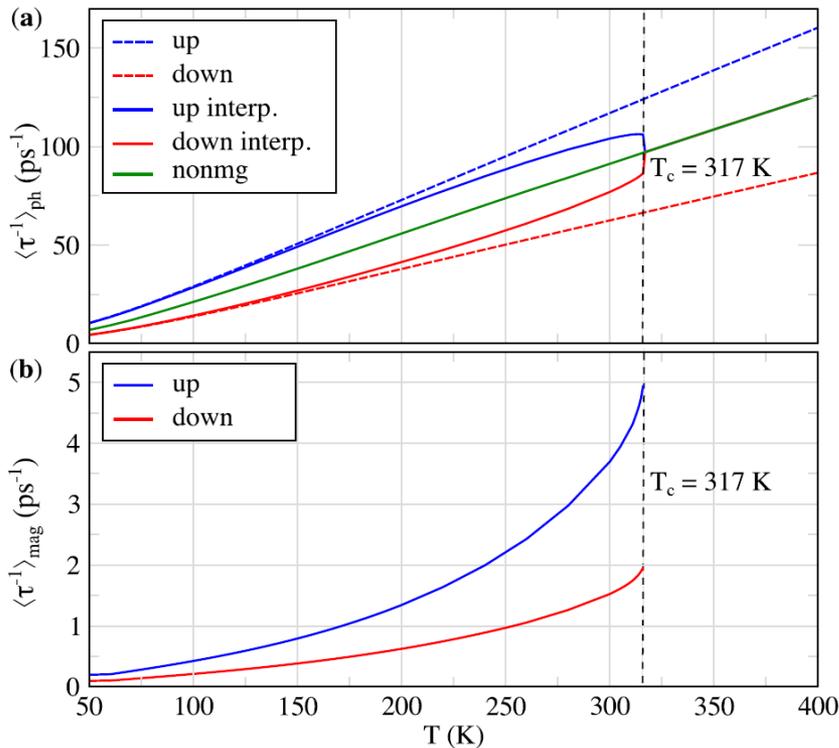
Lattice effects and magnetism in Fe₃GeTe₂ IV

Transport properties

$$\sigma_{xx}^{\sigma} = -\frac{e^2}{\Omega} \sum_{n\mathbf{k}} \frac{\partial f_{n\mathbf{k}}^{\sigma}}{\partial \varepsilon_{n\mathbf{k}}^{\sigma}} \tau_{n\mathbf{k}}^{\sigma} [V_{n\mathbf{k}\sigma}^x]^2$$

$$1/\tau_{n\mathbf{k}}^{\sigma} = \frac{2}{\hbar} \text{Im} \Sigma_{n\mathbf{k}}^{\sigma}$$

$$V_{n\mathbf{k}\sigma}^x = \partial \varepsilon_{n\mathbf{k}}^{\sigma} / \partial (\hbar k_x)$$



$$\Sigma_{n\mathbf{k}}^{\sigma}(\omega, T) = \sum_{m\mathbf{q}\nu} |g_{mn,\nu}^{\sigma}(\mathbf{k}, \mathbf{q})|^2 \times \left[\frac{b_{\mathbf{q}\nu} + f_{m\mathbf{k}+\mathbf{q}}^{\sigma}}{\omega - \varepsilon_{m\mathbf{k}+\mathbf{q}}^{\sigma} + \hbar\omega_{\mathbf{q}\nu} - i\eta} + \frac{b_{\mathbf{q}\nu} + 1 - f_{m\mathbf{k}+\mathbf{q}}^{\sigma}}{\omega - \varepsilon_{m\mathbf{k}+\mathbf{q}}^{\sigma} - \hbar\omega_{\mathbf{q}\nu} - i\eta} \right]$$

$$g_{mn,\nu}^{\sigma}(\mathbf{k}, \mathbf{q}) = \sqrt{\frac{\hbar}{2m_0\omega_{\mathbf{q}\nu}}} \langle \psi_{m\mathbf{k}+\mathbf{q}}^{\sigma} | \partial_{\mathbf{q}\nu} V^{\sigma} | \psi_{n\mathbf{k}}^{\sigma} \rangle$$

and similar for magnons

$$\Sigma_{n\mathbf{k}}^{\uparrow}(\omega, T) = 2I^2 \langle S_z \rangle \sum_{\mathbf{q}\nu} \frac{b_{\mathbf{q}\nu} + f_{n\mathbf{k}+\mathbf{q}}^{\uparrow}}{\omega - \varepsilon_{n\mathbf{k}+\mathbf{q}}^{\uparrow} + \hbar\omega_{\mathbf{q}\nu} - i\eta}$$

$$\Sigma_{n\mathbf{k}}^{\downarrow}(\omega, T) = 2I^2 \langle S_z \rangle \sum_{\mathbf{q}\nu} \frac{b_{\mathbf{q}\nu} + 1 - f_{n\mathbf{k}-\mathbf{q}}^{\uparrow}}{\omega - \varepsilon_{n\mathbf{k}-\mathbf{q}}^{\uparrow} - \hbar\omega_{\mathbf{q}\nu} - i\eta}$$

$$I = \frac{1}{2SN_F} \sum_{m\mathbf{k}\sigma} (\varepsilon_{m\mathbf{k}}^{\uparrow} - \varepsilon_{m\mathbf{k}}^{\downarrow}) \frac{\partial f_{m\mathbf{k}}^{\sigma}}{\partial \varepsilon_{m\mathbf{k}}^{\sigma}}$$

Dependence of phonon contribution on magnetization
Is much more important than direct magnon scattering

Lattice effects and magnetism in Fe_3GeTe_2 V

Phonon-induced renormalization of exchange interactions in metallic two-dimensional magnets

Danis I. Badrtdinov , Mikhail I. Katsnelson , and Alexander N. Rudenko *

PHYSICAL REVIEW B **110**, L060409 (2024)

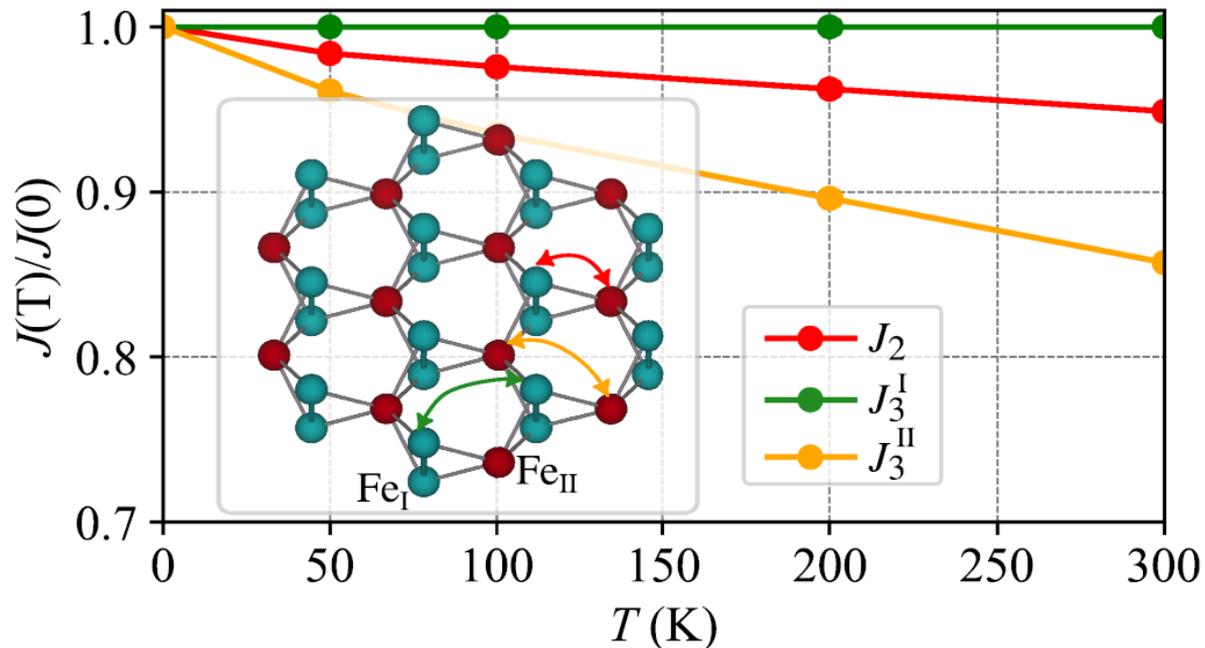
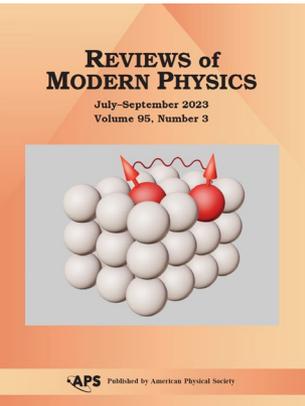
Magnetic force theorem for exchange in correlated systems

$$J_{ij} = 2\text{Tr}_{\omega L}(\Sigma_i^s G_{ij}^\uparrow \Sigma_j^s G_{ji}^\downarrow)$$

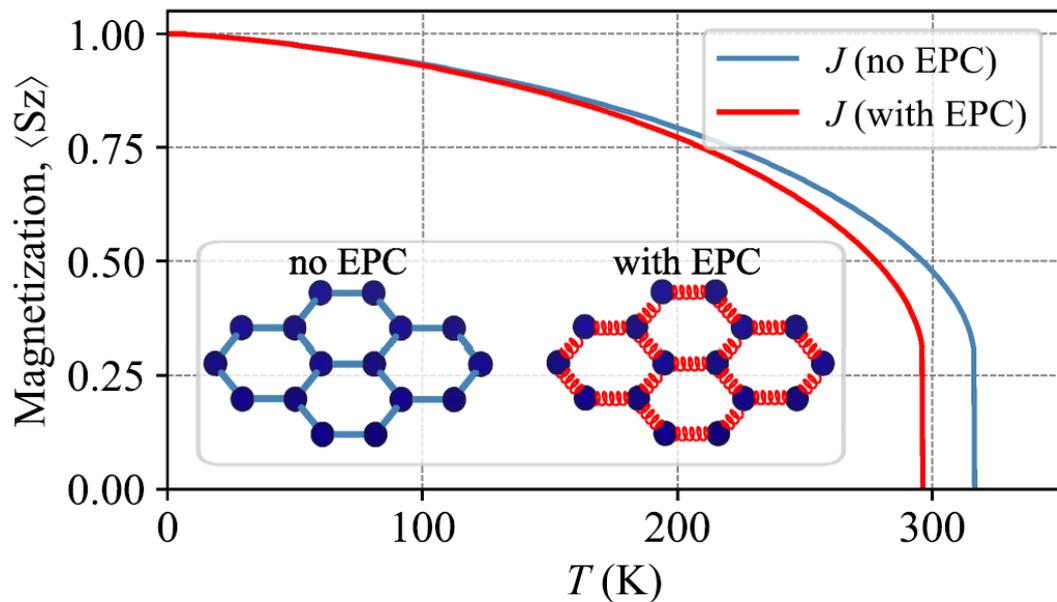
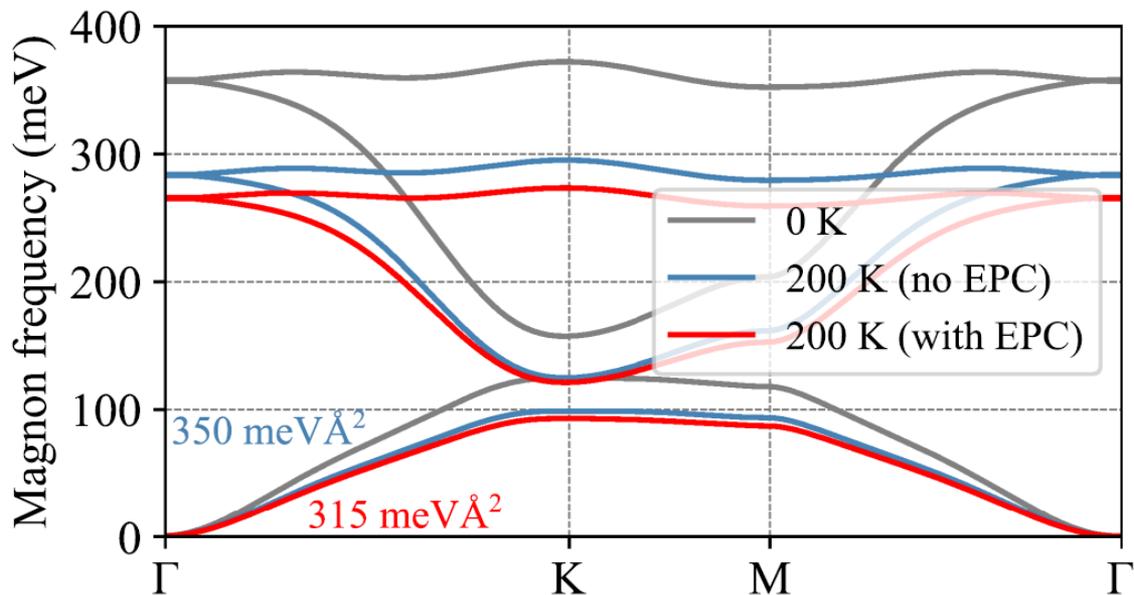
MIK & Lichtenstein 2000, for review see our recent review

$$\Sigma_i = \Sigma_i^c + \vec{\Sigma}_i^s \vec{\sigma}$$

The idea (very simple and straightforward!): include electron-phonon contributions into the self-energy – much simpler than explicitly do calculations for distorted systems



Lattice effects and magnetism in Fe_3GeTe_2 VI



Curie temperature decreased by
approx. 10%

Conclusions

- Known 2D magnets are strongly correlated, go beyond DFT (but GW-like methods seem to be sufficient)
- Be careful with dipole-dipole interactions, they are not just shape anisotropy
- Importance of p-d hybridization in CrX_3 , different magneto-optical properties for $\text{X} = \text{Br}$ and $\text{X} = \text{I}$
- Opportunity of electron self-trapping
- Tunability of magnetic properties via substrate
- Highly anisotropic CrSBr , very special optics sensitive to magnetism
- Nontrivial interplay of electron-magnon and electron-phonon interactions in 2D metallic magnets, phonons are important!

Many thanks to all collaborators and many thanks
to you for your attention